ЗАДАЧИ КЪМ ТРИГОНОМЕТРИЧНИ УРАВНЕНИЯ

Решете уравненията:

1. $\sin x \sin 3x + \cos 4x = 0$.

2. $\cos\left(x - \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{3}\right) + \cos 2x = 1$.

3. $8\sin^2 x - 7\cos^2 x = 8$.

4. $\sin 3x + \sin 5x = \sin 4x$.

5. $\sin 7x - \sin 3x = \sin 2x$.

6. $\cos x - 2\cos 3x + \cos 5x = 0$.

7. $\cos 5x + \cos 7x = \cos x$.

8. $\cos 3x - \sin 2x - \cos x = 0$.

9. $\cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x + \frac{\pi}{6}\right) = \cos\left(x - \frac{\pi}{4}\right)$.

10. $\cos x + \sin\left(2x + \frac{\pi}{6}\right) - \sin\left(2x - \frac{\pi}{6}\right) + 1 = \sqrt{3}\left(1 + 2\cos x\right)$.

11. $2\cos\left(\frac{x}{3} + \frac{\pi}{8}\right) - 2\cos\left(\frac{x}{6} - \frac{\pi}{12}\right) = \sqrt{3}\cos\left(\frac{x}{2} - \frac{\pi}{16}\right) + 3\sin\left(\frac{x}{12} - \frac{\pi}{16}\right)$.

12. $\sin^4 x + 5\cos 2x + 4 = 0$.

13. $4\sin x + \sqrt{3}\sin 2x = 2\cos 2x \sin x$.

14. $2\sin^2\left(5x+\frac{\pi}{4}\right)=1+\sin(2x+3)+2\sin^2\left(4x-\frac{3}{2}\right)\cos\left(6x+\frac{3}{2}\right)$.

15. $2\operatorname{tg}^2 x+4\cos^2 x=7$.

16. $\cos(2x+3)+\sin 9x=\sin^2\left(\frac{\pi}{4}+\frac{3x}{2}\right)-2\cos^2\left(\frac{3}{2}-5x\right)$.

17. $9\operatorname{cotg}^2 x+4\sin^2 x=6$.

18. $8\sin^4 x-13\cos 2x=7$.

19. $\sin^2 6x+5\sin^2 3x=2$.

20. $2\cos^4 x+1=3\cos 2x$.

21. $\sin^8 x-\cos^8 x=\frac{1}{2}\left(\cos^2 2x-\cos 2x\right)$.

22. $\cos^6 x-\sin^6 x=\frac{13}{8}\cos^2 x$.

23. $\sin^3 x+\cos^3 x=1+\frac{1}{2}\cos\left(2x+\frac{9\pi}{2}\right)$.

24. $\operatorname{tg}2x-4\sin x\cos x=4\sin^2 x$.

25. $4\sin^2 x+2\sqrt{3}\sin 2x+\frac{\sqrt{3}}{\operatorname{tg}2x}=5$.

26. $\left(1+\operatorname{tg}^2 2x\right)\left(\sin\frac{21x}{4}\cos\frac{7}{4}x+\sin\frac{5x}{4}\cos\frac{x}{4}\right)=$

$=\left(\sin\frac{x}{4}\cos\frac{5x}{4}-\sin\frac{7x}{4}\cos\frac{21x}{4}\right).\frac{1}{\cos^2 2x}$.

27. $3^{\frac{\operatorname{tg}2x}{2}}-\frac{3\sqrt{3}}{3^{\operatorname{tg}2x}}=0$.

28. $2^{\cos 2x}=\frac{1}{2^{\cos 2x+1}}$.

29. $2^{\sin 2x}-\frac{1}{2.2^{\sin 2x}}=0$.

30. $\sqrt{5}+5^{\frac{1}{2}+\log_5\sin x}=15^{\frac{1}{2}+\log_{15}\cos x}$.

31. $3^{-\frac{1}{2}}+6^{-\frac{1}{2}+\log_6\sin x}=2^{-\frac{1}{2}+\log_2\cos x}$.

32. $2-6^{\frac{1}{2}+\log_6\sin x}=2^{\frac{1}{2}+\log_2\cos x}$.

33. $\left|\operatorname{tg}x\right|=\operatorname{tg}x-\frac{1}{\cos x}$.

34. $|\cos x| = \cos x - 2\sin x$.

35. $|\operatorname{ctg} x| = \operatorname{ctg} x + \dfrac{1}{\sin x}$.

36. $1+2|\cos x|\sin x = 0$.

37. $4|\sin x| + 2\cos 2x = 3$.

38. $\left(\dfrac{3}{4} + 6\sin^2\dfrac{x}{2}\sin\left(\dfrac{x}{2}+\dfrac{3\pi}{2}\right)\right)^{\frac{1}{2}} = \sqrt{3}\cos\dfrac{3x}{4}$.

39. $\sqrt{\dfrac{1-4\cos^2 3x}{8\cos\left(2x-\dfrac{2\pi}{3}\right)}} = \cos\left(2x-\dfrac{\pi}{6}\right)$.

40. $\sqrt{5-2\sin x} = 6\sin x - 1$.

41. $\sqrt{10-18\cos x} = 6\cos x - 2$.

42. $\sqrt{37-48\operatorname{ctg} x} = 8\operatorname{ctg} x - 5$.

43. $\sqrt{13-18\operatorname{tg} x} = 6\operatorname{tg} x - 3$.

44. $\sqrt{3+4\sqrt{6}-\left(16\sqrt{3}-8\sqrt{2}\right)\cos x} = 4\cos x - \sqrt{3}$.

45. $\sqrt{9-4\sqrt{3}-\left(16-8\sqrt{3}\right)\sin x} = 4\sin x - 3$.

46. $\sqrt{9-6\sqrt{2}-\left(12-8\sqrt{2}\right)\cos x} + 2\sqrt{2} = 2\sqrt{2}\cos x + 1$.

47. $\sqrt{2+\sqrt{6}-\left(6\sqrt{2}-2\sqrt{3}\right)\sin x} = 2\sin x - \sqrt{2}$.

48*. $\sqrt[4]{3}\operatorname{tg} x - 1 = \left(\sqrt[4]{3}-1\right)\sqrt{\operatorname{tg} x}$.

49. $2\sin x - \sqrt[4]{3} = \left(2-\sqrt[4]{12}\right)\sqrt{\sin x}$.

50. $\sqrt[4]{8}\cos x - 1 = \left(\sqrt{2}-\sqrt[4]{2}\right)\sqrt{\cos x}$.

51. $\operatorname{tg} x - \sqrt[4]{3} = \left(\sqrt[4]{3}-1\right)\sqrt{\operatorname{tg} x}$.

52. $\sqrt{2-3\cos 2x} = \sqrt{\sin x}$.

53. $\sqrt{1+4\cos 2x} = \sqrt{1-4\cos x}$.

54. $\sqrt{3+4\cos 2x} = \sqrt{2\cos x}$.

55. $\sqrt{1-4\sin x} = \sqrt{1-4\cos 2x}$.

56. $\sqrt{5\sin x + \cos 2x} + 2\cos x = 0$.

57. $\sqrt{2\cos x\sin 2x}=\sqrt{5\sin x+4\sin 2x}$.

58. $\sqrt{5\cos x-\cos 2x}+2\sin x=0$.

59. $\sqrt{2\sin x\sin 2x}=\sqrt{5\cos x+4\sin 2x}$.

60*. $\log_{\frac{1}{3}}\left(\sin\frac{x}{2}+\cos 2x\right)+\log_3\left(\sin\frac{x}{2}-\sin x\right)=0$.

61. $\log_5\left(\cos\frac{x}{2}+3\operatorname{tg}x-\frac{3\sqrt{3}}{2}\right)=\log_5\left(\cos\frac{x}{2}+\operatorname{tg}2x-\frac{3\sqrt{3}}{2}\right)$.

62. $\log_{\frac{1}{6}}\left(\sin\frac{x}{2}-3\operatorname{tg}x-\frac{3\sqrt{3}}{2}\right)+\log_6\left(\sin\frac{x}{2}-\operatorname{tg}2x-\frac{3\sqrt{3}}{2}\right)=0$.

63. $\frac{\sin x}{(x-4)^2}+|\sin x|=0$.

64. $(x-2)^2|\cos x|=\cos x$.

65. $\left(x+\frac{1}{2}\right)^2|\sin x|+\sin x=0$.

66. $\sqrt{9-x^2}\,(2\sin 2\pi x+5\cos\pi x)=0$.

67. $\sqrt{25-4x^2}\,(3\sin 2\pi x+8\cos\pi x)=0$.

68. $\sqrt{49-4x^2}\left(\sin\pi x+3\cos\frac{\pi x}{2}\right)=0$.

69. $\frac{\sin 2x}{1+\sin x}=-2\cos x$.

70. $\frac{\sin x}{1+\cos x}=1-\cos x$.

71. $2\sin x=\frac{\sin 2x}{1-\cos x}$.

72. $\frac{1-\cos x}{\cos\left(\frac{x+\pi}{2}\right)}=2$.

73. $\frac{1+2\sin^2 x-3\sqrt{2}\sin x+\sin 2x}{2\sin x\cos x-1}=1$.

74. $\frac{\sin 2x}{\sin 3x}=1$.

75. $\dfrac{\cos x\left(3\sqrt{2}-2\cos x\right)-\sin 2x-1}{2\sin x\cos x-1}=-1.$

76. $\dfrac{\sin 2x}{\cos 3x}=-1.$

77. $\dfrac{2-3\sin x-\cos 2x}{6x^2-\pi x-\pi^2}=0.$

78. $\dfrac{\cos 2x-2\cos^2 x-2}{12x^2-8\pi x+\pi^2}=0.$

79*. $\sin x=x^2+x+1.$

80. $2\cos^2\dfrac{x^2+x}{6}=2^x+2^{-x}.$

81. $\sin^6 x+\cos^{10} x=1.$

82. $\sqrt{\sin^5 x}+\sqrt{\cos^5 x}=\sqrt[4]{2}.$

83. $\sin x+\sin 9x=2.$

84. $\cos x-\cos\dfrac{x}{2}=2.$

85. $\cos\pi x=x^2-4x+5.$

ОТГОВОРИ

1. $x=\frac{\pi}{6}+n\frac{\pi}{3}$. **2.** $x=n\pi,\quad x=(-1)^k\frac{\pi}{6}+k\pi$. **3.** $x=\frac{\pi}{2}+n\pi$. **4.** $x=n\frac{\pi}{4}, x=\pm\frac{\pi}{3}+2k\pi$. **5.** $x=\pm\frac{\pi}{30}+\frac{2n\pi}{5}+n\frac{\pi}{2}$. **6.** $x=\frac{\pi}{6}+n\frac{\pi}{3}, x=k\pi$. **7.** $x=\frac{\pi}{2}+n\pi, x=\pm\frac{\pi}{18}+k\frac{\pi}{3}$. **8.** $x=n\frac{\pi}{2}, x=(-1)^k\frac{\pi}{6}+k\pi$. **9.** $x=\frac{3\pi}{4}+n\pi$. **10.** $x=\pm\frac{2\pi}{3}+2n\pi$.

11. $x=-\frac{\pi}{12}-(-1)^n\frac{4\pi}{3}+4n\pi, x=-\frac{5\pi}{4}+12k\pi$. **12.** $x=\frac{\pi}{2}+n\pi$. **13.** $x=n\pi, x=\pm\frac{5\pi}{6}+2k\pi$. **14.** $x=\frac{(2n+1)\pi-3}{12}, \quad x=\frac{(8k+1)\pi+3}{8}$. **15.** $x=\pm\frac{\pi}{3}+n\pi$.

16. $x=\frac{\pi}{12}+\frac{n\pi}{6}, \quad x=\frac{(4k-1)\pi+6}{14}, \quad x=3-\frac{3\pi}{2}+m\pi$. **17.** $x=\pm\frac{\pi}{3}+n\pi$.

18. $x=\pm\frac{\pi}{6}+n\pi$. **19.** $x=\pm\frac{\pi}{18}+\frac{n\pi}{3}$. **20.** $x=n\pi$. **21.** $x=\frac{\pi}{4}+n\frac{\pi}{2}, \ x=\frac{\pi}{2}+k\pi$.

22. $x=\frac{\pi}{4}+n\frac{\pi}{2}, x=\pm\frac{\pi}{6}+k\pi$. **23.** $x=2n\pi, x=\frac{\pi}{2}+2k\pi$. **24.** $x=\frac{\pi}{8}+n\frac{\pi}{2}, x=\pm\frac{\pi}{6}+k\pi$.

25. $x=(-1)^n\frac{\pi}{6}+n\frac{\pi}{2}, \quad x=\frac{\pi}{12}+k\frac{\pi}{2}$. **26.** $x=n\frac{\pi}{2}, \ x=\frac{\pi}{6}+k\frac{\pi}{3}$. **27.** $x=\frac{\pi}{8}+n\frac{\pi}{2}$.

28. $x=\pm\frac{\pi}{3}+n\pi$. **29.** $x=(-1)^{n+1}\frac{\pi}{12}+n\frac{\pi}{2}$. **30.** $x=\frac{\pi}{6}+2n\pi$. **31.** $x=\frac{\pi}{12}+2n\pi$.

32. $x=\frac{\pi}{12}+2n\pi$. **33.** $x=\frac{5\pi}{6}+2n\pi$. **34.** $x=\frac{\pi}{4}+(2n+1)\pi$. **35.** $x=\frac{2\pi}{3}+2n\pi$.

36. $x=-\frac{\pi}{4}+2n\pi, \quad x=\frac{5\pi}{6}+2k\pi$. **37.** $x=\pm\frac{\pi}{6}+n\pi$. **38.** $x=\pm\frac{4\pi}{3}+4(2n+1)\pi$.

39. $x=n\pi$. **40.** $x=(-1)^n\frac{\pi}{6}+n\pi$. **41.** $x=\pm\frac{\pi}{3}+2n\pi$. **42.** $x=\operatorname{arctg}\frac{4}{3}+n\pi$.

43. $x=\operatorname{arctg}\frac{2}{3}+n\pi$. **44.** $x=\pm\frac{\pi}{4}+2n\pi$. **45.** $x=(-1)^n\frac{\pi}{3}+n\pi$. **46.** $x=\pm\frac{\pi}{4}+2n\pi$.

47. $x=(-1)^n\frac{\pi}{3}+n\pi$. **48.** $x=\frac{\pi}{4}+n\pi$. Положете $\sqrt{\operatorname{tg}x}=u\geq 0$. **49.** $x=(-1)^n\frac{\pi}{6}+n\pi$.

50. $x=\pm\frac{\pi}{4}+2n\pi$. **51.** $x=\frac{\pi}{3}+n\pi$. **52.** $x=(-1)^n\frac{\pi}{6}+n\pi$. **53.** $x=(2n+1)\pi$.

54. $x=\pm\frac{\pi}{3}+2n\pi$. **55.** $x=-\frac{\pi}{2}+2n\pi$. **56.** $x=\frac{5\pi}{6}+2n\pi$. **57.** $x=n\pi, \ x=\frac{2\pi}{3}+2k\pi$.

58. $x=-\frac{\pi}{3}+2n\pi$. **59.** $x=\frac{\pi}{2}+n\pi$. **60.** $x=-\frac{\pi}{6}+2n\pi, \ x=\frac{7\pi}{6}+4k\pi$. Уравнението е

равносилно със системата $\begin{vmatrix} \cos 2x = -\sin x \\ \sin\frac{x}{2} - \sin x > 0 \end{vmatrix}$. Получаваме $\sin x = 1$ (не удоволетворява неравенството) и $\sin x = -\frac{1}{2}$. Сега от решенията на $\sin x = -\frac{1}{2}$ трябва да отделите тези, за които неравенството е изпълнено. **61.** $x = \frac{\pi}{6} + 4n\pi$.

62. $x = \frac{5\pi}{6} + 4n\pi$. **63.** $x = 5, x = n\pi$. **64.** $x = 1, \quad x = \frac{\pi}{2} + n\pi$. **65.** $x = -\frac{3}{2}, x = n\pi$.

66. $x = \pm 3, x = \pm\frac{5}{2}, x = \pm\frac{3}{2}, x = \pm\frac{1}{2}$. **67.** $x = \pm\frac{5}{2}, x = \pm 2, x = \pm 1, x = 0$. **68.** $x = \pm\frac{7}{2}$, $x = \pm 3, x = \pm 1$. **69.** $x = \frac{\pi}{2} + 2n\pi, \quad x = (-1)^{k+1}\frac{\pi}{6} + k\pi$. **70.** $x = 2n\pi, x = \frac{\pi}{2} + 2k\pi$.

71. $x = (2n+1)\pi, \quad x = \pm\frac{\pi}{3} + 2k\pi$. **72.** $x = 3\pi + 4n\pi$. **73.** $x = \frac{3\pi}{4} + 2n\pi$.

74. $x = \frac{\pi}{5}(2k+1), \quad k \neq 5n+2$. **75.** $x = \frac{3\pi}{4} + 2n\pi$. **76.** $x = \frac{3\pi}{10} + \frac{2n\pi}{5}, n \neq 5k+3$.

77. $x = (-1)^n\frac{\pi}{6} + n\pi, \quad x = \frac{\pi}{2} + 2k\pi, \quad k \neq 0$. **78.** Няма решение. **79.** Няма решение (начертайте графиките на $\sin x$ и на $x^2 + x + 1$ и докажете, че те не се пресичат). **80.** $x = 0 \quad (2^x + 2^{-x} \geq 2)$. **81.** $x = k\frac{\pi}{2}$. **82.** $x = \frac{\pi}{4} + 2n\pi$. **83.** $x = \frac{\pi}{2} + 2n\pi$.

84. $x = 4n\pi$. **85.** $x = 2$.